**IND-secure quantum symmetric encryption based on point obfuscation**

**MOTIVATION**

We rigorously construct an IND-secure quantum symmetric encryption scheme by means of quantum point obfuscation. Our work formally demonstrates the implementation and application of quantum obfuscation. We hope that such work will be constructive in the field of quantum obfuscation.

**BACKGROUND**

Quantum symmetric encryption

In the IND-security game, adversary $A$ constructs the message $m_{ME}$ and tries to distinguish if his message is encrypted. We denote the function of $A$ by separately quantum polynomial-time algorithm (QPT) $A = (M, D)$. $M$ is the message generator, while $D$ is the ciphertext distinguisher.

1. IND-security game

An IND-secure quantum symmetric encryption scheme is IND-CPA, if $A$ has oracle access to $Enc_x$.

Quantum obfuscation

- (Polynomial expansion) $m = \text{play}(n)$
- (Functional equivalence) for any possible $\rho$ $|\text{Pr}[D(O(C)) - U_{\text{pol}}(U)]| \leq \text{negl}(n)$
- (Virtual black-box) for every QPT $A$, there exists a QPT $S^\infty$ such that for any QPT distinguisher $D$ $|\text{Pr}[D(O(C))] - 1| \leq |\text{Pr}[D(S^\infty(C))]| = 1 \leq \text{negl}(n)$.

A quantum point function $U_{a,\beta}$ with a general output is $U_{a,\beta}: |0^n\rangle \rightarrow |x, P_{a,\beta}(x)\rangle$, where $a \in \{0,1\}^n$, $\beta \in \{0,1\}^m$, and $P_{a,\beta}$ is a classical function point with a multi-bit working as:

$$P_{a,\beta}(x) = \begin{cases} \beta & \text{if } x = a \\ 0^m & \text{otherwise} \end{cases}$$

**PROPOSED SCHEMES**

1. Quantum point obfuscation with a general output

A quantum point function $U_{a,\beta}$ with a general output is

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We give the construction of the obfuscator $O$ and the interpreter $\delta$ and then prove polynomial expansion, functional equivalence, and virtual black-box property of them. The obfuscator $O$ works as follows. The obfuscator $O$ randomly generates $n$ bits of classical string $\omega$ and then queries the quantum random oracle with $U_{\omega}$. $O$ computes $|\text{Pr}[\|\omega\|\omega\|\|\beta\|\beta]|$. The output of the obfuscation of $C_{a,\beta}$ is

$$O(C_{a,\beta}) = |\omega, \|\omega\|\omega\|\|\beta\|\beta\rangle$$

2. Overall construction of the interpreter $\delta$

**SCHEME ANALYSIS**

1. IND-CPA-secure encryption from a self-combinable obfuscator

If a quantum point obfuscator $O$ is self-combinable, then the quantum symmetric encryption scheme is IND-CPA-secure.

$$\begin{aligned} &|\text{Pr}[D(\text{Enc}_{\text{ME}}(\text{rand}(k))))| = 1 \\
&= \text{Pr}[D(\text{Enc}_{\text{ME}}(\text{rand}(k))))] = 1 \leq \text{negl}(n) \end{aligned}$$

2. Leakage-resistant resilient from an obfuscation with auxiliary input

If $(\alpha, \beta)$ is an quantum point obfuscator with auxiliary input $f$, then the quantum symmetric encryption scheme is leakage-resistant against key information $f(k)$.

$$\begin{aligned} &|\text{Pr}[D(\text{Enc}_{\text{ME}}(\text{rand}(k))))] = 1 \\
&= \text{Pr}[D(\text{Enc}_{\text{ME}}(\text{rand}(k)))) = 1 \leq \text{negl}(n) \end{aligned}$$

**CONCLUSION**

To develop the theory of quantum obfuscation, its application is crucial. In this paper, we demonstrated the usability of a quantum point obfuscator in quantum symmetric key encryption. We gave the construction of quantum point obfuscation and proposed an IND-secure quantum symmetric encryption scheme based on point obfuscation. Then we proved the corresponding relationship between IND-security and obfuscator for a quantum symmetric encryption. In particular, different encryption security can be implemented according to the properties of an obfuscator. While previous works on encrypting quantum data are either based on QOTP or lacking implementation, our work gives the first concrete construction of quantum data encryption with reusable private key. Further work lies in how to build a suitable obfuscator by means of quantum encryption schemes.