Dissipative dynamics of quantum states in the fiber channel

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In this work we consider the Liouville equation that describes the quantum non-unitary dynamics of quantum states in optical fiber. We consider particular case of thermalization aiming to applications related to various quantum information protocols; however the model can be generalized in various ways taking into account more features (external pump, nonlinear interactions, continuous spectra, free space propagation, etc.). In order to obtain the appropriate evolution models for states in the channel we use the SU(1,1) algebra formalism in the Liouville representation. Considered implementation of model takes into account dichroism, retardance, thermalization, dispersion, decoherence in polarization domain. Described approach allows to connect the information properties of quantum channels with its physical ones. In order to illustrate this statement we consider BB84 quantum key distribution protocol and investigate behavior of quantum bit error rate affected by considered physical phenomena in optical fiber. <u>(Paper will be available soon at PhysRev A)</u>

Introduction

Any quantum state (except equilibrium one) is subject to decoherence due to its connection with the environment. The latter nonunitary dynamics may alter information properties of propagating in the optical fiber quantum optical states in a nontrivial way [1]. Nonunitary dynamics may be described by Lindblad scheme [2]. The goal of the research is to develop the model of dissipative dynamics of quantum states and study its relations with dynamics of quantum information protocols' characteristics.

Liouville equation

We would like to consider the following Liouville equation that describes quantum non-unitary dynamics of arbitrary state ρ_0 for single optical mode with frequency ω influenced by thermal light [2]:

$$\frac{\partial}{\partial t}\rho(t) = -i[H, \rho(t)] + \Gamma\rho(t),$$

$$\rho(t)|_{t=0} = \rho_0,$$

where

$$H = \omega\left(a^\dagger a + \frac{1}{2}\right),$$

$$\Gamma\rho = -\frac{\gamma}{2}\Big((n+1)\left(a^\dagger a\rho + \rho a^\dagger a - 2a\rho a^\dagger\right) + n\left(aa^\dagger\rho + \rho aa^\dagger - 2a^\dagger\rho a\right)\Big),$$

$$n = \frac{1}{e^{\frac{\hbar\omega}{kT}} - 1},$$

where $a^\dagger$ and a are creation and annihilation operators, ω is optical frequency, γ is thermalization rate, n is mean thermal photon number for temperature of the environment $T(K)$, $\hbar$ is reduced Planck constant, k is Boltzmann constant.

We introduce Liouville notation as follows:

$$A\rho \equiv \overleftarrow{A}\rho, \qquad \rho A = \overrightarrow{A}\rho,$$

and the following operators:

$$K_+ = \overleftrightarrow{a^\dagger}\vec{a}, \qquad K_- = \overleftarrow{a}\overrightarrow{a^\dagger},$$

$$K_0 = \frac{1}{2}\left(\overleftarrow{a^\dagger a} + \overrightarrow{a^\dagger a}\right), \quad N = \overleftarrow{a^\dagger a} - \overrightarrow{a^\dagger a}.$$

Operators K_+, K_-, and K_0 form SU(1,1) algebra with the following properties:

$$[K_0, K_\pm] = \pm K_\pm, \qquad [K_-, K_+] = 2K_0.$$

Then one may introduce Liouvillian L constructed with the latter operators as the solution of initial differential equation:

$$\rho(t) = e^{Lt}\rho_0 = e^{\frac{n}{n+1}K_+}A(t)e^{\frac{-n}{n+1}K_+},$$

$$A(t) = e^{-(n+1)K_-}e^{L^{(d)}t}e^{(n+1)K_-},$$

$$L^{(d)} = -i\omega N - \gamma\left(K_0 - \frac{1}{2}\right),$$

Where $L^{(d)}$ is diagonalized Liouvillian. Then assuming n small we use Baker–Campbell–Hausdorff expression and obtain the following relation:

$$e^{Lt} \approx A(t) + \frac{n}{n+1}[K_+, A(t)].$$

The final step is to avoid nonlinear dependence of $A(t)$ on n by introducing another operator:

$$B(t) = e^{-K_-}e^{L^{(d)}t}e^{K_-},$$

$$A(t) = B(t) - n[K_-, B(t)].$$

We present the approximate solution of Liouville equation as follows:

$$\rho(t) = \left(I + n(1 - e^{-\gamma t})(K_+ - 2K_0 + K_-)\right)B(t)\rho_0,$$

where I is identity operator.

References:

1. Nielsen M. A., Chuang I. Quantum computation and quantum information. – 2002.
2. Carmichael H. An open systems approach to quantum optics: lectures presented at the Université Libre de Bruxelles, October 28 to November 4, 1991. – Springer Science & Business Media, 2009. – T. 18.
3. Klyshko D. M. Polarization of light: Fourth-order effects and polarization-squeezed states //Journal of Experimental and Theoretical Physics. – 1997. – T. 84. – №. 6. – C. 1065-1079.
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Stokes parameters and quantum bit error rate

Further we consider two-mode problem. Then by multiplying both sides of equation with appropriate Stokes operator [3] and taking trace one may derive system of equation for Stokes parameters as follows:

$$\frac{\partial}{\partial t}S_0(t) = -\left(\frac{\gamma_H + \gamma_V}{2}\right)S_0 - \left(\frac{\gamma_H - \gamma_V}{2}\right)S_1 + n(\gamma_H + \gamma_V),$$

$$\frac{\partial}{\partial t}S_1(t) = -\left(\frac{\gamma_H - \gamma_V}{2}\right)S_0 - \left(\frac{\gamma_H + \gamma_V}{2}\right)S_1 + n(\gamma_H - \gamma_V),$$

$$\frac{\partial}{\partial t}S_2(t) = -\left(\frac{\gamma_H + \gamma_V}{2}\right)S_2 - \left(\frac{\omega_H - \omega_V}{2}\right)S_3,$$

$$\frac{\partial}{\partial t}S_3(t) = -\left(\frac{\omega_H - \omega_V}{2}\right)S_2 - \left(\frac{\gamma_H + \gamma_V}{2}\right)S_3.$$

For simplicity we consider only diagonal elements of frequency and relaxation matrices. Despite this fact dynamics of Stokes parameters for considered rather simple model takes into account dichroism, polarization mode dispersion, thermalization, and as the consequence of the latter decoherence in polarization domain.

Let us consider BB84 quantum key distribution protocol [4] with the following initial states with respect to some chosen basis (H, V):

$$|H\rangle = |1\rangle_H \otimes |0\rangle_V, \quad |H\rangle = |0\rangle_H \otimes |1\rangle_V,$$

$$|S\rangle = \frac{|H\rangle + |V\rangle}{\sqrt{2}}, \qquad |F\rangle = \frac{|H\rangle - |V\rangle}{\sqrt{2}}.$$

Without doubt one may assume that polarization mode dispersion can be compensated, i.e. $\omega_H = \omega_V$. Then each of equiprobable states participates in the emergence of quantum bit error rate as follows:

$$Q_H = \frac{1 - \left|\frac{S_1^H(t)}{S_0^H(t)}\right|}{2}, \qquad Q_V = \frac{1 - \left|\frac{S_1^V(t)}{S_0^V(t)}\right|}{2},$$

$$Q_S = \frac{1 - \left|\frac{S_2^S(t)}{S_0^S(t)}\right|}{2} + \frac{\left|\frac{S_1^S(t)}{S_0^S(t)}\right|}{2}, \quad Q_F = \frac{1 - \left|\frac{S_2^F(t)}{S_0^F(t)}\right|}{2} + \frac{\left|\frac{S_1^F(t)}{S_0^F(t)}\right|}{2},$$

Where total quantum bit error can be found as follows:

$$Q = \frac{Q_H + Q_V + Q_S + Q_F}{4}.$$

In figure below dependence of total quantum bit error Q on normalized propagation time

$$t_Q = \ln\left(\frac{1}{2n}\right)\frac{1}{\max(\gamma_H, \gamma_V)}$$

for $\frac{\gamma_H}{\gamma_V} = 0.97$, $\omega_H = \omega_V$, and $n = 10^{-13}$ is shown. As one may notice there are two main contributions to the shape of considered curve. The first one is linear slope; it is due to dichroism:

$$Q_{LS} = \frac{|\gamma_H - \gamma_V|t}{4}.$$

The second one is due to thermalization (and depolarization):

$$Q_{WD} = \frac{n(1 - e^{-\gamma t})}{2n(1 - e^{-\gamma t}) + e^{-\gamma t}},$$

assuming $\gamma = \gamma_H = \gamma_V$.