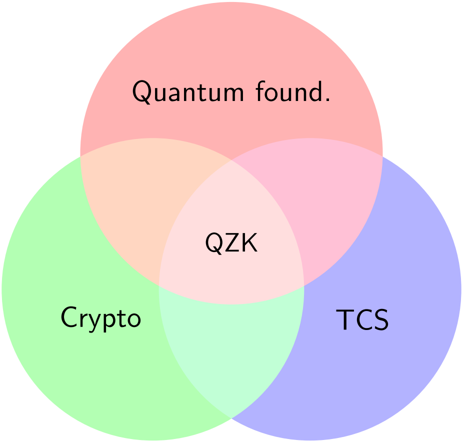
Quantum zero-knowledge from Locally Simulatable Proofs

Alex Bredariol Grilo

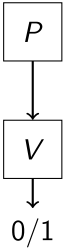
joint work with Anne Broadbent (U. of Ottawa)
arxiv:1911.07782


Quantum found.
QZK
Crypto
TCS

Interactive proofs

Interactive proofs

$L \in \mathrm{NP}$



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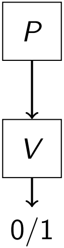

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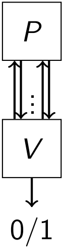

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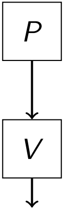

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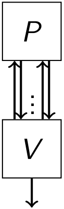
$L \in \mathrm{IP} = \mathrm{PSPACE}$



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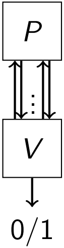
Zero-knowledge



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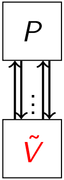

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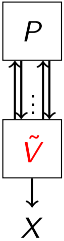

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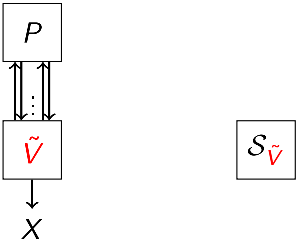

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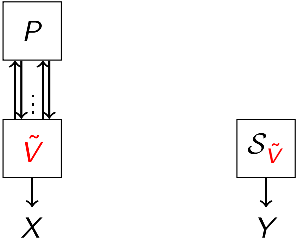

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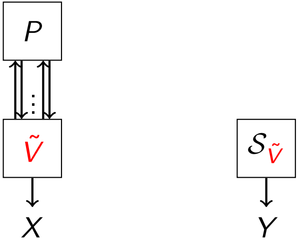

Computational zero-knowledge

X and Y cannot be **efficiently** distinguished:

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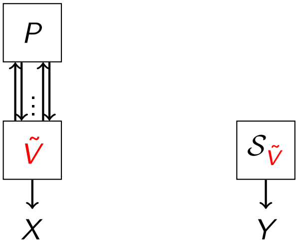

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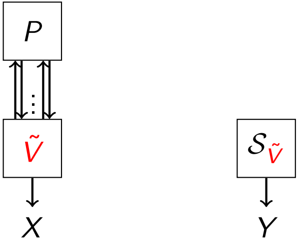

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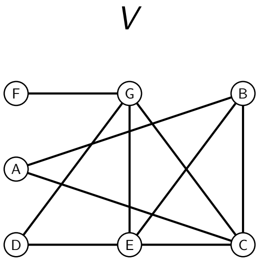
Fundamental notion in modern cryptography!

Example: ZK for 3-coloring



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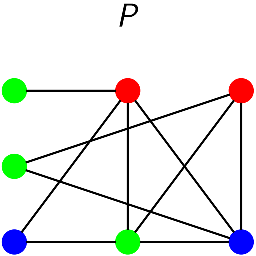

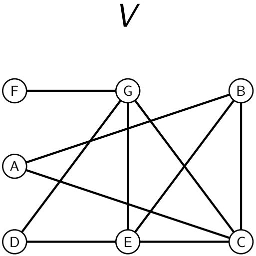

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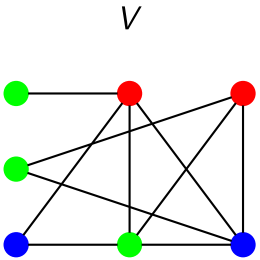

Example: ZK for 3-coloring

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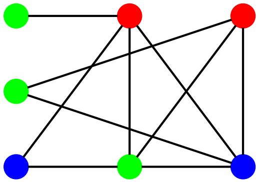

Completeness ✓ Soundness ✓ ZK ✗

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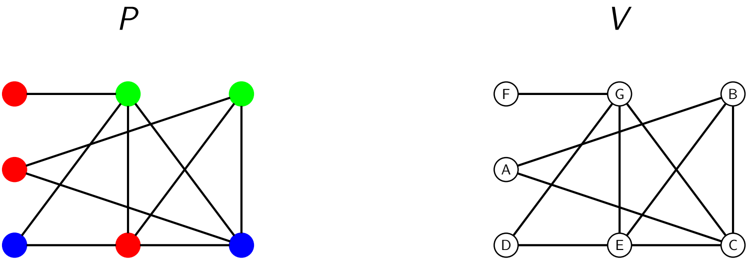

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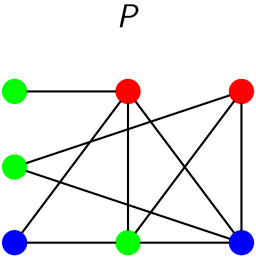

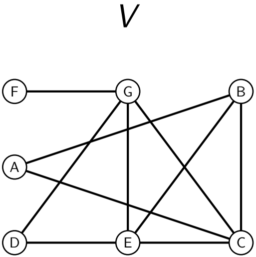

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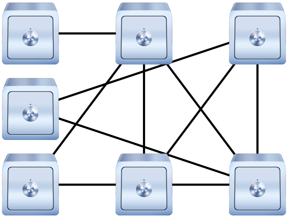

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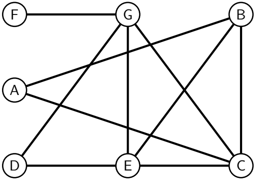

Example: ZK for 3-coloring

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$A \rightarrow 564651$
$B \rightarrow 867132$
$C \rightarrow 984565$
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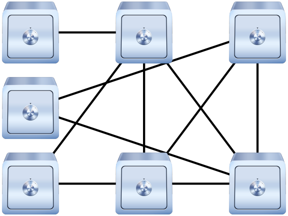

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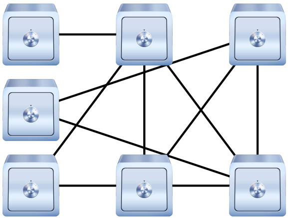

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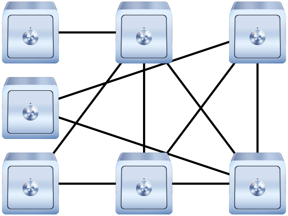

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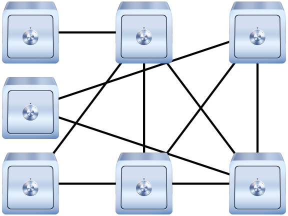

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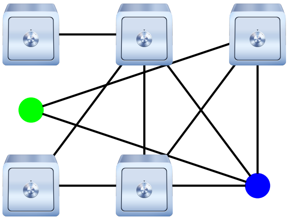

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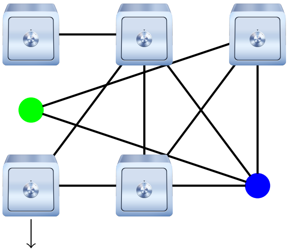

bit-commitment

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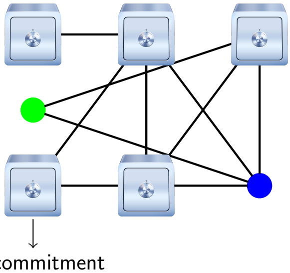

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Quantum proofs

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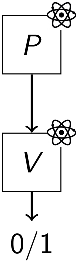

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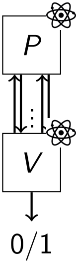

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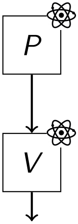

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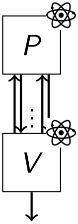

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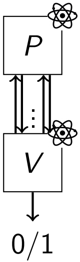

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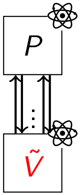

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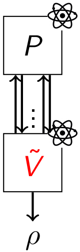

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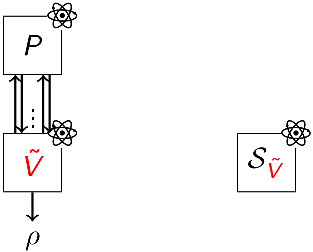

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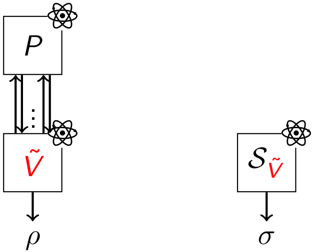

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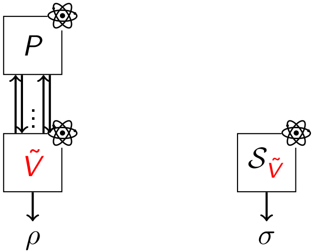

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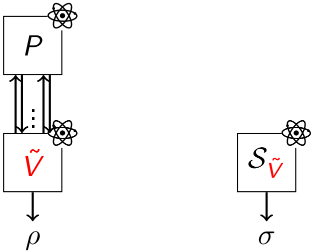

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    Need to go through QMA $\subseteq$ PP

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  Multiple rounds of communication

  Somewhat complicated

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  - ⋆ QNISZK for QMA in the secret parameters setup

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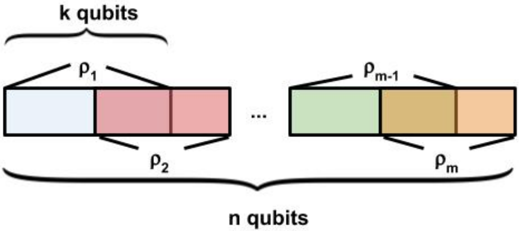
Input: Reduced density matrices $\rho_1, ..., \rho_m$ on k-qubits

Output: yes: $\exists \psi$ such that $\forall i$: $\left\| Tr_{\overline{S_i}}(\psi) - \rho_i \right\| \leq \varepsilon$

no: $\forall \psi$, $\exists i$: $\left\| Tr_{\overline{S_i}}(\psi) - \rho_i \right\| \geq \frac{1}{poly(n)}$



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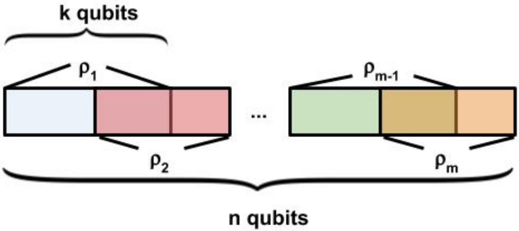

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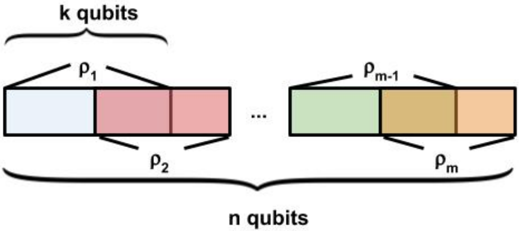

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Very simple ZK proof for QMA

P V

$\rho_1, \ldots, \rho_m$

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P $\qquad\qquad$ V

$\psi^{\otimes \ell}$ $\qquad\qquad$ $\rho_1, \ldots, \rho_m$

Very simple ZK proof for QMA

P

V

$X^a Z^b \psi^{\otimes \ell} Z^b X^a$

$\rho_1, \ldots, \rho_m$

a_1, b_1

a_2, b_2

$\ldots$

a_{n-1}, b_{n-1}

a_n, b_n

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$X^a Z^b \psi^{\otimes \ell} Z^b X^a$ $\rho_1, \ldots, \rho_m$

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$\dots$
$a_n, b_n \rightarrow 894102$

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Very simple ZK proof for QMA

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$a_n, b_n \to 894102$

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Completeness ✓ Soundness ✓ ZK ✓

Simulatable codes - Steane code

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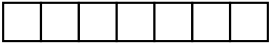

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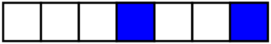

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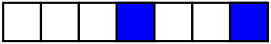

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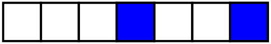

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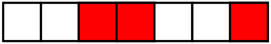

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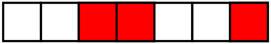

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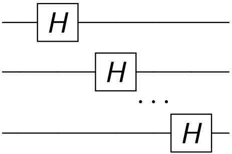
   - *transversal Clifford gates*
   - T*-gadgets*

$Enc(\rho)$



CLDM is QMA-hard

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Goal

Tweak the verification algorithm such that we can compute the reduced density matrices of history states.

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Encoded circuit

Instead of $V = U_T \ldots U_1$ and initial state $|\psi_{init}\rangle$, consider the circuit V' that

1. Receives $\frac{1}{2^n} \sum_{a,b} Enc(|a,b\rangle \langle a,b| \otimes X^a Z^b |\psi\rangle \langle\psi| Z^b X^a)$
2. Check encoding of the witness
3. Undoes the OTP of the witness
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Theorem

There is a classical simulator that computes in polynomial time the reduced density matrices of the history state of the encoded verifier. Moreover there is a global state consistent with the reduced density matrices iff it is a yes-instance.

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3. There is a polynomial-time algorithm that computes the density matrices of the history state
   - Most of clock qubits are traced-out, so the remaining state is a mixture of intervals

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- Properties of (ZK) interactive proof system

  Completeness: there is a good strategy for yes-instance

  Soundness: there is no good strategy for no-instance

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[1] Independent concurrent work by Coladangelo, Vidick and Zhang.

Open questions

- Find applications for QZK
- $\mathsf{MIP}^{ns} = \mathsf{PZK}\text{-}\mathsf{MIP}^{ns}$?
- QNIZK protocol for QMA in the CRS model
- QMA-hardness of (bosonic) representability [LCV'07, WMN'10], universal functional of density function theory [SV'09]

Thank you for your attention!